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# Mathematics News Letter

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Dedicated to mathematics in general and to the following aims in particular: (1) a study of the common problems of secondary and collegiate mathematics teaching, (2) a true valuation of the disciplines of mathematics, (3) the publication of high class expository papers on mathematics, (4) the development of greater public interest in mathematics by the publication of authoritative papers treating its cultural, humanistic and historical phases.



VOL. 7 BATON ROUGE, LA., DECEMBER, 1932 No. 3

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## MISSISSIPPI AND LOUISIANA MATHEMATICIANS MEET AT RUSTON MARCH 3, 4

This issue of the NEWS LETTER carries a message from Chairman C. D. Smith of the Louisiana-Mississippi Section of M. A. of A. to the mathematics teachers of the two States. The joint once-a-year meeting of the Section with the Louisiana-Mississippi Branch of the National Council of Mathematics Teachers is dated to be held at Ruston, Louisiana, March 3, 4. The Chairman's announcement will be noted with keen interest by NEWS LETTER readers. A revised feature of the usual Section program is the voluntary offering of papers by those who have results of research or of teaching to report. This revision is to be commended.

Mathematical workers who are recent comers into our Louisiana-Mississippi territory should be reminded that the definite correlation of the secondary and the collegiate mathematical forces in these two States in the form of joint annual conventions for the study of interlocking teaching problems, is of many years standing. It should be gratifying to all true friends of the cause of advancing mathematical science to know that this "Louisiana-Mississippi plan" has been observed with marked interest by mathematical groups in various parts of the country—some of whom have commended it as a model for adoption.—S. T. S.

# THE OPERATIONS ON SIGNED NUMBERS

By WILSON L. MISER

The operations of addition, subtraction, multiplication, and division as they are performed in arithmetic apply only to the numerical values of signed numbers. In these operations on signed numbers there are the signs of quality as well as the numerical or absolute values to be taken into account. It is the purpose of this paper to give definitions of the four fundamental operations on signed numbers. The definitions will be stated in such ways that they include the operations of arithmetic as special cases. Each operation will be defined in a geometrical way by representing the signed numbers by directed line segments.

**Addition.** Let the signed numbers a and b be represented by line segments OA and OB, respectively, as in Fig. 1.

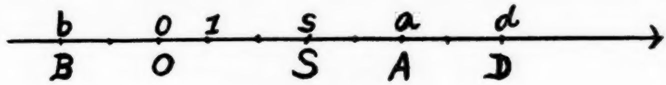


Fig. 1.

To add b to a is equivalent to adding OB to OA. Therefore, from the endpoint A of OA, draw the line segment AS of the same length as that of OB and in the same direction as that of OB. Then the line segment OS represents the signed number which is the sum of a and b.

For example, in figure 1 $a = 5$, $b = -2$, and $s = 3$.

**Subtraction.** To subtract b from a is equivalent to subtracting OB from OA. Therefore, from the endpoint A of OA, draw the line segment AD of the same length as that of OB but in the OPPOSITE direction to that of OB. The line segment OD represents the signed number d which is the difference a minus b.

For example, in figure 1 $a = 5$, $b = -2$, and $d = 7$.

Subtraction appears in the definition as the inverse of addition, for the subtraction of OB from OA is performed by reversing the direction of OB and then proceeding as in addition.

From these definitions it is easy to prove:

(1) The addition of a negative number to another number is equivalent to subtracting a positive number of the same absolute value.

(2) The subtraction of a negative number from another number is equivalent to adding a positive number of the same absolute value.

**Multiplication.** Let the lines MX and NY in figure 2 be perpendicular to each other at the point 0, the origin or zero point of each line. Take the line segment OU on OY to represent a unit in length. Consider the axes MX and NY directed and numbered just as the coordinate axes are in rectangular coordinates for plotting graphs.

Let a and b denote two signed numbers. To multiply a by b, represent a on MX by OA and b on NY by OB. Draw the line UA, and, from the point B, draw the line BP parallel to the line UA. Then the line segment OP represetns the number p which is the product ab.

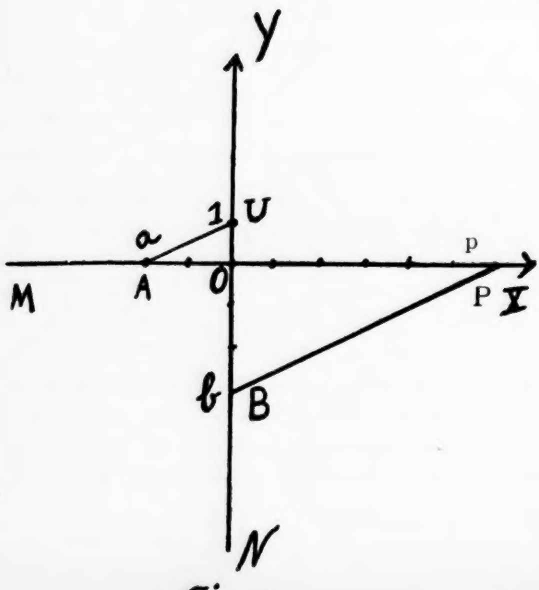


Fig. 2.

For example, in figure 2 $a = -2$, $b = -3$, and $p = 6$. This is an example of a negative number multiplied by a negative number. The definition gives the products for a positive number by a positive, a positive by a negative, and a negative by a positive. It is easy to show that $ab = ba$, the commutative law.

**Division.** To divide a by b, represent a on MX by OA and b on NY by OB as in figure 3. Draw the line AB, and, from the point U, draw the line UQ parallel to the line AB. Then the line segment OQ represents the number q which is the quotient a/b.

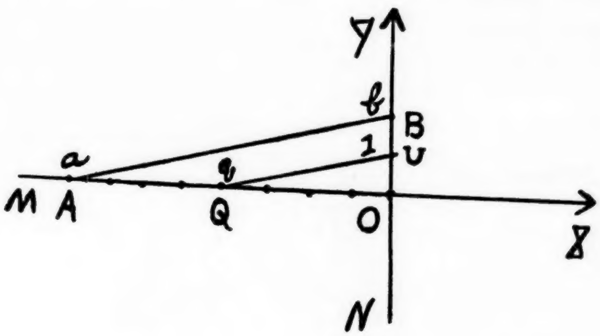


Fig. 3.

For example, in figure 3 $a = -8$, $b = 2$, and $q = 4$. This is an example of a negative number divided by a positive number. The definition gives the quotients of a positive number divided by a positive, a positive divided by a negative, and a negative divided by a negative.

It is easy to prove in figure 2 that

$$OP = OA \cdot OB$$

and in figure 3 that

$$OQ = OA/OB.$$

These proofs are made by simple theorems of plane geometry.

The author has tested out the definitions as given here in the classroom many times and has found them very useful in making clear that there are indeed the four operations on signed numbers and that each operation is a single one although it appears from the rules in common use that each operation is made of two or more.

# ANNOUNCEMENT

January 11, 1933.

To the Mathematics Teachers of
Mississippi and Louisiana.

Dear Fellow Teachers:

We are now able to give you the following preliminary announcement regarding the annual meeting of the Section and the Council. The place is L. P. I., Ruston, Louisiana and the time is March 3 and 4.

I am able to quote the chairman on local arrangements with the following important information. Rooms are available on the Campus without cost if notice is given in advance. You should write Mr. Henry Schroeder, Ruston, Louisiana, for room reservation at least a week in advance. Hotel rates are:

One in room \$2.00 per day, private bath.
Two in room \$2.50 per day, private bath.
One in room \$1.50 per day, connecting bath.
Two in room \$2.00 per day, connecting bath.

We will have a visiting scientist for the special lecture at the open session Friday evening after which the annual banquet will be held. A strong program will be given and matters of special importance will come before the Association. The opportunity to report briefly on special problems will be a special feature of the regular program. Such topics should be sent to the secretary Dr. Deborah May Hickey, Cleveland, Mississippi, at once.

The important thing is your presence at the session. The benefits of this meeting will far outweigh the cost to the teacher. In fact the above financial arrangements render the cost a matter of negligible importance. This will be the largest session in the history of the Section if you are there. Your Committee is making all effort for the success of this meeting with the firm belief that you will not fail us. Meet us in Ruston on the third of March.

Sincerely yours,

C. D. SMITH,
Section Chairman.

# INTRINSIC DECIMALS

By JAMES McGIFFERT
Rensselaer Polytechnic Institute

Many are familiar with the peculiarity of the repeating decimal,

$$.142857142857$$

which is the decimal value of one-seventh.

It is easily seen that

$$\frac{2}{7} = .285714285714$$

$$\frac{3}{7} = .428571428571$$

$$\frac{4}{7} = .571428571428$$

$$\frac{5}{7} = .714285714285$$

$$\frac{6}{7} = .857142857142$$

It is seen that each of these repeating decimals contains the same figures, in exactly the same order, but with a different beginning figure in the sequence.

If we learn the succession of figures .142857, we have in these six figures the values of all sevenths. We need merely to find the first

figure, by division, and then proceed in the sequence from that point.

Thus $\frac{4}{7}$ begins with 5, since 7 goes into 40 between five and six times.

and we at once write off the decimal .571428, from the same sequence.

In various periodicals this decimal has been discussed, and called a magic number, belonging exclusively to the number 7, perhaps because 7 has for many years been called a perfect number.

The same property, however, is possessed by all prime numbers. If the number of figures in the repetend is less by unity than the number whose repeater it is, the decimal is called a Perfect Repeater, which implies that in it are contained all the repeaters for all the proper fractions with this same denominator. Thus we have for 17, the following repeater,

$$\frac{1}{17} = .0588235294117647\underline{0588235294117647}$$

In the repeater here written there are seen to be 16 figures, and as $17-1=16$, we have here another perfect repeater. Hence all seventeenths are contained in this one set of figures. We have for example,

$$\frac{3}{17} = .1764705882352941\underline{1764705882352941}$$

Here we chose the second figure 1, since 30 is nearly twice 17. Of course the practical value of this long decimal is not great, because it appears to most persons a bit arduous to commit to memory a long succession of figures. But a bit of practice makes this exercise easy.

The reason why we call the repeaters in these two cases perfect repeaters is because there are only $n-1$ proper fractions with $n$ as their denominator.

Repeaters for fractions with composite denominators may be obtained by factoring the denominators, and we consider only prime numbers in this discussion of repeaters.

The reader may enjoy the exercise of finding the intrinsic decimals for all prime numbers, if he contemplates living for an infinite number of years, for his own interest and edification.

Let us now consider some special repeaters. The number 13 presents a semi-repeater, as is evident, because

$$\frac{1}{13} = .076923\underline{076923}$$

and

$$\frac{2}{13} = .153846\underline{153846}$$

Each of these repeaters contains 6 figures, which is the half of 12, thus constituting each as a semi-repeater.

These two repeaters must therefore contain all the possible thirteenths. Trial will show that in the repeater for $\frac{1}{13}$ are contained those for 3, 4, 9, 10, and 12 thirteenths, while the repeater for $\frac{2}{13}$ contains those for all the other thirteenths.

It will be found that the repeater for 31 is also a semi-repeater, containing 15 figures in the repetend. We have

$$\frac{1}{31} = .032258064516129\underline{032258064516129}$$

This repeater fits $\frac{2}{31}$, as will readily be seen, but not $\frac{3}{31}$, whose value is

$$\frac{3}{31} = .096774193548387\underline{096774193548387}$$

The reader will have no difficulty in determining which of these repeaters contains any desired number of thirty-firsts.

The numbers 23 and 29 furnish Perfect Repeaters, but the number 37 presents a strange set of decimals.

$$\frac{1}{37} = .027\underline{027}$$

As this repeater contains only three figures, there must be 12 repeaters for 37. By trial these may readily be found. The 36 proper fractions with 37 as denominator, will be found to arrange themselves in 12 sets of three figures each, thus verifying our statement.

Thus we see that 1, 10, and 26 thirty-sevenths contain the same sequence of figures, namely, .027, 270, and 702. Similar results will be obtained for all other thirty-sevenths.

Thus we see what strange freaks there seem to be in the various repeaters for the prime numbers.

The writer suggests that it might be a good subject for research to endeavor to find out why 7, 17, 19, 23, 29, etc. furnish perfect repeaters, while 3, 13, 31, 37, etc., do not.

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## ON SIMULTANEOUS ALGEBRAIC EQUATIONS

By H. L. SMITH
Louisiana State University

In the last issue of the MATHEMATICS NEWS LETTER, Professor Miser gave a treatment of simultaneous algebraic equations based on Sylvester's dyalitic method of elimination. The method, in the case of two equations in two unknowns, was to eliminate y and solve the resulting equation for x, then eliminate x from the original equations and solve the resulting equation for y, and finally to pair

each value of x with every value of y and to accept or reject each such pair after actual substitution into the original system.

It is plain that this method gives all the solutions of the system and it is to be expected that it will also usually give extraneous solutions. The following question suggests itself: Will values of x ever be found which cannot be paired with any of the values of y to give a solution? The answer is Yes. We show this by an example and also show by that example how to modify the method so as to avoid testing for extraneous solutions.

Let the equation to be solved be

$$(1) \qquad (x-1)y^2+xy+3=0,$$

$$(x^2-1)y^2+x^2y-1=0.$$

On multiplying each of these equations by y, adjoining the resulting equations to the system and forming the determinant of the coefficients of $y^3, y^2, y, 1$ in the resulting system, we get

$$\begin{vmatrix} x-1, & x, & 3, & 0 \\ 0, & x-1, & x, & 3 \\ x-1, & x, & -1, & 0 \\ 0, & x-1, & x, & -1 \end{vmatrix} = 0,$$

which reduces to

$$(2) \qquad (x-1)(x+1)(3x^2+4x-8)=0.$$

Now instead of eliminating x by Sylvester's method we do so by substituting each value of x from (2) into (1) and consider the resulting system.

If $x=1$, the system (1) reduces to

$$y+3=0, \; y-1=0$$

which is inconsistent. Hence $x=1$ cannot correspond to a solution of (1).

If $x=-1$, then (1) becomes

$$2y^2+y-3=0, \quad y-1=0$$

The left members of these equations have the H. C. F. $y-1$ and hence the equations have the common solution $y=1$. Hence (1) has the solution $x=-1$, $y=1$. We here avoided an extraneous solution $x=-1$, $y=-{}^3/_2$ which would have been obtained had we eliminated x by Sylvester's method.

Now let $x=a$, where a is one of the roots of $3x^2+4a-8=0$.

Then

$$3a^2=-4a+8,$$

and (1) reduces by aid of this fact to

$$\text{(3)} \qquad \begin{aligned}&(a-1)y^2+ay+3=0\\&(-4a+5)y^2+(-4a+8)y-3=0\end{aligned}$$

Let the left members of (3) be denoted by f,g respectively and let us determine the H. C. F. of f and g. This we do by slightly modifying Euclid's algorithm.

We have

$$4f+g=y^2+8y+9.$$

Hence any common factor of f and g is a factor of $y^2+8y+9$. We also have

$$f=(a-1)(y^2+8y+9)-[(7a-8)y+(9a-12)]$$

$$g=(-4a+5)(y^2+8y+9)+4[(7a-8)y+(9a-12)]$$

Hence any common factor of f,g must be a factor of $(7a-8)y+(9a-12)$, and it also follows that this quantity will itself be a common factor (and indeed the H. C. F.) if $y^2+8y+9$ is divisible by it. By actually

dividing $(7a-8)^2(y^2+8y+9)$, written in the form $[(7a-8)y]^2+8(7a-8)[(7a-8)y]+9(7a-8)^2$, by $(7a-8)y+(9a-12)$ we get $(7a-8)y+6(7a-8)+(5a-4)$ as quotient and $6(3a^2+4a-8)$ as remainder. But $3a^2+4a-8=0$. Hence $(7a-8)y+(9a-12)$ is the H. C. F. of f and g, and (3) is equivalent to

$$(7a-8)y+(9a-12)=0.$$

This equation has the solution

$$y=(12-9a)/(-8+7a) \tag{4}$$

If we represent by $\bar{a}$ the root other than a of $3x^2+4x-8=0$, then

$$a+\bar{a}=-4/3,\ a\bar{a}=-8/3 \tag{5}$$

and (4) becomes on multiplying numerator and denominator both by

$-8+7\bar{a}$,

$$y=(-96+72a+84\bar{a}-63a\bar{a})/[64-56(a+\bar{a})+49a\bar{a}]$$

$$=\frac{3}{2}(6+6a+7\bar{a}),$$

on using (5). Thus

$$x=a,\ y=\frac{3}{2}(6+6a+7\bar{a}) \tag{6}$$

is a solution of (1) and by symmetry

$$x=\bar{a},\ y=\frac{3}{2}(6+6\bar{a}+7\bar{a}) \tag{7}$$

is another.

The quadratic formula shows that we may take

$$a = \frac{2}{3}(-1+\sqrt{7}),\ a = \frac{2}{3}(-1-\sqrt{7})$$

Then (6), (7) become

$$x = \frac{2}{3}(-1+\sqrt{7}),\ y = -4-\sqrt{7};$$

$$x = \frac{2}{3}(-1-\sqrt{7}),\ y = -4+\sqrt{7},$$

respectively, which together with $x = -1$, $y = 1$ are all the solutions of (1).

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## CERTAIN PROBLEMS

By GASTON S. BRUTON
The University of the South

Every teacher of mathematics from time to time wanders from the subject assigned. This is a practice which probably is to be condemned if the new topic is utterly foreign to mathematics. However, every one has found that the introduction of new material or an example not in the lesson may be helpful in holding the attention of the students and in creating interest. Most textbooks give historical notes, and these can and should be supplemented by the instructor. A student feels better acquainted with Cardan's Formulas if he knows about the mathematical duel between Tartaglia and Cardan. It is easy to digress from mathematics to logic or philosophy. Mathematical induction is purely an exercise in logic. One of my freshmen said that he suspected his difficulty in grasping mathematical induction was due to the fact that it was too logical for him to follow.

Most students have difficulty with "written"problems in algebra. I have found that the average student is better able to analyze such

problems if he looks upon them as examples in translation from one language to another. Just as he thinks "I do not know" when he sees "Je ne sais pas", I try to get him to think "$x+y=20$" when he sees "The sum of two numbers is twenty". Applied problems are presumed to interest the student, but many of these problems are so detailed that he gets lost and loses interest in that which is supposed to create interest. I believe it probably better to tell him about some of the applications and here astronomy is a fertile and interesting field. The transition from a study of conic sections to a talk on astronomy is easy and beneficial. For example, I have yet to see a class of freshmen who were not interested in how the Leonids came into our solar system and how they changed their orbit from a parabola to an ellipse.

In addition to such standard subjects which are constantly referred to in teaching a course in mathematics, each instructor has his own methods and examples which he employs either to illustrate some point or to revive a sleepy class. The main object of this paper is to give a few examples that I have collected from time to time. I have made little attempt to show under what conditions they may be used, for these conditions depend on the circumstances and on the individual class.

I have made use of the following three examples to illustrate a number of things such as to show what difficulties we may get into by performing our operations blindly or how carelessness may lead to absurdities.

1. To prove $2=1$.

Let $a=b$.

Then $a^2-b^2=a^2-ab$,

or $(a-b)(a+b)=a(a-b)$.

Thus $a+b=a$,

or $a+a=a$,

or $2a=a$,

or $2=1$.

2. To prove $-1=+1$, or $3=1$.

$$\sqrt{-1}=\sqrt{-1}.$$

As $$-1=\frac{-1}{1}=\frac{1}{-1},$$

then $$\sqrt{\frac{-1}{1}}=\sqrt{\frac{1}{-1}}$$

or $$\frac{\sqrt{-1}}{\sqrt{1}}=\frac{\sqrt{1}}{\sqrt{-1}}$$

Thus by cross multiplication

$$(\sqrt{-1})^2=(\sqrt{1})^2,$$

or $$-1=+1,$$

or $$1=3.$$

3. To prove that a flea is as heavy as an elephant.

Let a be the weight of the elephant, and b be the weight of the flea.

Let $$\frac{a+b}{2}=c.$$

Then $$a+b=2c.$$

If we multiply both sides by $a-b$, then

$$a^2-b^2=2ac-2bc.$$

Thus $$a^2-2ac=b^2-2bc.$$

Adding $c^2$ to both sides,

$$a^2-2ac+c^2=b^2-2bc+c^2,$$

or $$(a-c)^2=(b-c)^2,$$

or $$a-c=b-c.$$

Thus $$a=b.$$

All of us have heard students give a proof by saying "Why that is obvious". The following examples show that frequently the obvious answer is not the correct one.

4. How many quarter-inch cubes can be made from an inch cube?

Very few students will guess sixty-four.

5. A has two loaves of bread and B has three. C gives A and B together a dime and the three of them eat equal quantities of bread. How should A and B divide the dime?

The obvious answer is four cents and six cents whereas the correct answer is two cents and eight cents.

6. A car goes a mile up-hill and then a mile down-hill. The speed up-hill is thirty miles per hour. What must be the rate down-hill if the average speed for the two miles is to be sixty miles per hour?

The obvious answer is ninety miles per hour even though the problem is impossible.

7. A straight railroad track is one mile long. A siding built along the arc of a circle is one mile and three feet long. How far apart will the two tracks be at their mid-points?

The answers to this problem vary from a fraction of an inch to a foot. The correct answer is approximately 64.4 ft.

The following problems appear superficially to be impossible to solve, but are not beyond freshmen of more than average ability.

8. A man wishes to buy 100 articles for 100 dollars. There are three kinds, the first kind costing fifty cents, the second five dollars and the third ten dollars. How many of each kind must he buy? Answer: 90, 9, 1.

9. A boy has a certain number of apples and on his way home passes through three toll gates. At the first gate he leaves half his apples and half an apple more. At the second gate he leaves half his remaining apples and half an apple more, and does the same at the third gate. He has one apple left and has not split an apple. How many apples did he start with? 15.

10. Three hunters caught a certain number of rabbits. Next morning A got up at six o'clock and divided the rabbits into three equal piles with one rabbit left over which he gave to the dog. A took his share and went to town. B got up at seven o'clock and not knowing what A had done divided the rabbits into three equal piles with one left over which he threw away. He took his share and left. C repeated the process at eight o'clock. The three men met in town and discovering what been done went back to the camp and divided the remaining rabbits into three equal piles with one left over. How many rabbits were there at first?

This problem was given me by a freshman in algebra who felt that he should have been able to solve it but found that he could not. The problem, of course, is indeterminate, and the smallest answer is 79.

Many other topics which have the same beneficial results as the problems mentioned above may be introduced. I am listing a few of these in the form of questions.

Why is the series $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+ - - -$ called the Harmonic Series? Where did the name "algebra" come from? What is the oldest mathematical record? What is the Pythagorean representation of the regular solids? What book has more editions than any other besides the Bible? Why did the Dutch Astronomer Huyghens after having found one satellite of Saturn not find some of the others? How was it that Tartaglia discovered Cardan's Formulas? How was the planetoid Ceres lost and found? How was Neptune discovered? What is the curve of least time? Of natural growth? Of natural de-

cay? What is the probability curve? How was it found? How did Zeno prove there was no motion? How can there be as many points on a two-inch line as on a one-inch line? How can there be as many even numbers as there are even and odd together? Can anything have only one surface? Take an empty tobacco can and fill it with water then pour in a glass. Fill the can half-full and pour in the glass. Now put all the water back in the can. It can be done and furnishes a good illustration of maxima and minima problems.

What do the following men have in common? The philosophers Pythagoras, Plato, Aristotle, Des Cartes, Leibniz; the astronomers Hipparchus, Ptolemy, Clavius, Torricelli, Gallileo, Kepler; the physicists Archimedes, Roger Bacon, Newton, La Place, Huyghens; Thales the lawmaker; Omar Khayyam the poet; Durer the painter and sculptor; Bradwardine, Archbishop of Canterbury; Napier the Scotch minister; Oresme, a bishop in Normandy; C. L. Dodgson, minister, or better known as Lewis Carroll the author of *Alice in Wonderland.* They were all mathematicians.

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## NOTE ON A PROBLEM

By RICHARD MORRIS
Rutgers University

Problem 3470 [1931,50] was proposed by F. L. Wren in the Monthly for January, 1931, and a solution was given in the issue for October, 1931. In the Mathematics Teacher for December, 1928, the undersigned published an article on Stewart's Theorem with Applications. A formula for $\Sigma\, d_i^2$ is there given in the form

$$\frac{(n-1)}{2}(b^2+c^2)-\frac{(n^2-1)a^2}{6n},$$

from which comes the solution for the right triangle if $a^2=h^2$, and also the general solution, since $b^2+c^2=a^2-2bc\cos A$. In the Mathematics

News Letter for November, 1932, a still further generalization is pointed out by Professor W. V. Parker, which takes the form

$$\Sigma\,(a_i^2+b_i^2+c_i^2)=\frac{(n-1)(5n-1)}{6n}\,(a^2+b^2+c^2),$$

where $a_i$ is the length of a Stewart line from vertex A to side a. This last also comes immediately from my discussion of Stewart's Theorem.

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## UNIVERSITY OF WASHINGTON

### Department of Mathematics

### SEATTLE

January 17, 1933.

*Dear Professor Sanders*:

Supplementary to the bibliography on Humanizing Mathematics compiled by Professor Archibald, and published in the November, 1932, number of MATHEMATICS NEWS LETTER, I beg leave to submit the following titles:

In *Memorabilia Mathematica* (McMillan Company, New York).

**First**: Chapters entitled
Mathematics as a Fine Art.
Mathematics as a Language
Mathematics and Logic
Mathematics and Philosophy
Mathematics and Science

**Second**: Social Value of Mathematics—*Northwest Journal of Education*, December, 1910; January, 1911.

**Third**: Relation of Mathematics to Commerce—*School Science and Mathematics*, Volume 19, (1919).

**Fourth**: Mathematics and Efficiency in Secondary School Work—*School Science and Mathematics*, Volume 16 (1916).

**Fifth**: Mission of Modern Mathematics—*Scientific Monthly*, Volume 26, (1928).

These articles are all by the undersigned who finds it difficult to understand why all of these titles should have been ignored in the published list above referred to

Yours very sincerely,

ROBERT E. MORITZ.

REM:CM.

# *PRECOCITY IN MATHEMATICS

By S. T. SANDERS

As a rule, the poet, the musician, the artist manifest their particular bents at early ages. On the other hand, it is thought by some that talents which are predominantly intellectual begin to develop in relatively later years. History shows that in many cases mathematical ability is as precocious as that of the poet or the musician.

Henry Briggs, English mathematician, born in 1561, was made Gresham professor of mathematics at Cambridge at the age of 35.

Galileo began his researches at the age of 21. In 1589 when he was only 23 years old he was given the chair of mathematics at the University of Pisa.

Johann Kepler was born in 1571 and at the age of 22 was made professor of astronomy in the University of Grätz.

Cavalieri, Italian mathematician, assumed the mathematics chair at Bologna in 1629 when he was only 31.

Pascal, at the age of 14 was sitting at the weekly meetings of famous French geometers. At 16 he had written an essay on the conic sections.

John Wallis (1616-1703) used two weeks to master arithmetic when he was only 15 years old. At the age of 33 he was made Savilian professor of geometry at Oxford.

Huygens (1629-1695) was writing critical essays in mathematics by the time he was 22 years old.

Ball says: "Before Leibnitz was twenty he had mastered the ordinary text books on mathematics, philosophy, theology and law."
He was born in 1646 and died 1716.

Colin Maclaurin (1698-1746) was only nineteen when he was elected to the chair of mathematics at Aberdeen.

*Citations are from Ball's History of Mathematics.

Euler (1707-1783) was honored with the chair of mathematics in the University of Petrograd when he was twenty-six years of age. He succeeded Daniel Bernoulli, who had been appointed to this chair at the age of twenty-four.

Lagrange (1736-1813) was nineteen when he solved what was known as the "isoperimetrical problem"—a problem which had been discussed among mathematicians for over 50 years.

Karl Friedrich Gauss (1777-1855) laid the foundation for his vast achievements in mathematics before he was twenty. Much of his research in number theory was done before he left Carolina College.

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## PROBLEM DEPARTMENT

Edited by
T. A. BICKERSTAFF
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This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and solve problems here proposed.

Problems, and solutions will be credited to their authors.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

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No. 25. Proposed by E. C. Kennedy, College of Mines, El Paso, Tex.

Solve for least positive value of x to within an error of about one second or less. The solution may be obtained very easily if the expression is simplified properly.

$$\sqrt{\sin 2x} = \sqrt{2} \cos x \, [\sqrt[u]{4 \ . \ 76} - 1]$$

$$\text{Where } u = 1 + \sqrt{\tan x}$$

Solved by the proposer.

$$\sqrt{2\sin x\cos x} = \sqrt{2}\cos x[\sqrt[u]{4\ .\ 76} - 1]$$

$$\text{Where } u = 1 + \sqrt{\tan x}$$

$$\sqrt{\sin x} = \sqrt{\cos x}\ [\sqrt[u]{4\ .\ 76} - 1]$$

$$\sqrt{\tan x} = \sqrt[u]{4\ .\ 76} - 1$$

$$1 + \sqrt{\tan x} = \sqrt[u]{4\ .\ 76}$$

$$u = \sqrt[u]{4\ .\ 76}$$

$$u^u = 4\ .\ 76$$

$$\text{Whence,}\quad 2.10125 = u$$

[See *American Math. Monthly,* vol. xxxviii, No. 8.]

$$\sqrt{\tan x} = 1.10125$$

$$\tan x = 1.21275$$

$$x = 50°29'3''$$

Also solved by S. T. Sanders, Jr., St. Joseph Mo.

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**Problems for Solution**

No. 26. Proposed by T. A. Bickerstaff:

Find the x's which represent digits in the following sum of a geometric progression:

$$4x + xxx + xxx + xxxx + xxxxx = xxxx1$$

No. 27. Proposed by H. T. R. Aude, Colgate University:

Show that the equation

$$2xy - px - py = 0$$

where p is any prime number greater than 2 has three and only three positive integral solutions.

No. 28. Proposed by H. T. R. Aude, Colgate University:

Find the area in the xy-plane enclosed by the axes and

$$\cos^{-1}(x+y) - x - y = 0.$$

No. 29. Proposed by H. T. R. Aude, Colgate University:

Two circles each of radius r move so that their centers stay on a fixed line. Find the area A, common to the two circles, in terms of s the distance betweeen the centers, as s varies from 2r to 0.